the conditions specified in our experimental procedure.

The application of Schelkunoff's work which has been used in previous radial-line filter design[7] was investigated.

Attention was called to the fact that radial-line coaxial filters have a high return loss at frequencies well below their resonant frequency which makes them particularly appropriate for circuits commonly occurring in parametric work.

The waveguide harmonic filter shown in Fig. 11 has been utilized in parametric amplifier work[1] and due to its excellent signal performance has solved one of the filtering problems associated with waveguide parametric amplifiers.

Appendix

The derivation of the minimum loss ratio in Ragan[3] approximates relation (1) by

$$\lambda = \pi(a + b)$$

and then minimizes the coaxial-line loss subject to this relation. It is perhaps comforting to know that if a designer computes the multimoding limit of the sum $a+b$ from relation (1) and then picks some percentage of this as a practical design limit (*i.e.*, he will not work exactly at the point of multimoding) that the condition $(a+b)$ equal to a constant, when combined with the minimum loss requirement, leads to exactly the same ratio. Thus the minimum loss ratio becomes of practical import.

Acknowledgment

Thanks are due S. F. Jankowski who did the electroforming and many of the measurements. The author also wishes to express thanks to C. F. Edwards of the Holmdel laboratory for many informative discussions.

[7] "The Microwave Engineers Handbook," Horizon House—Microwave Inc., Brookline, Mass., pp. TD-49; 1961–62.

Periodic Cylinder Arrays as Transmission Lines*

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Summary—Periodic structures of conducting cylinders have been used as radiators (Yagi antennas), and, more recently, as slow-wave lines in traveling-wave tubes and masers. In this report it is shown that a nonresonant structure may have interesting capabilities as an open surface-wave transmission line. By means of a relatively simple matching network, efficient excitation of a surface wave on the periodic line is obtained. Response is flat over a 20 per cent frequency range at X band for several combinations of cylinder lengths and spacings. Total insertion losses are less than 3 db and largely independent of length of transmission line. Conducting cylinders are embedded in styrofoam.

The effects of bends and twists in the line have also been investigated. It is shown experimentally that a guided wave on this periodic structure can follow a circular path having 1.5λ radius of curvature with very little loss. The plane of polarization can be rotated 90° by inserting a short twisted section.

By terminating the transmission line with short circuits at both ends, a discrete series of transmission maxima is observed. Since these resonant peaks of transmission are of high Q factor, the dispersion characteristic of the line is obtained with very good accuracy.

This type of open transmission line may offer advantages over heavy-weight and bulky conventional waveguides for some specialized applications.

I. Introduction

A NUMBER OF infinitely long periodic structures theoretically support a propagating plane wave along their axes.[1,2,3] In practice, the guiding structures are of finite length, and any desired propagating mode has to be excited by a set of currents which can neither be infinite in amplitude nor can they be distributed over an infinite aperture in space. Another complication arises from the fact that the structure is terminated and we have also the reflected wave to consider. For these reasons, the performance one obtains experimentally is often substantially different from theoretical predictions. However, quite useful approximations may be obtained, and in many cases one is in a position to estimate the bounds of the error.

As a first step in calculating propagation characteristics and excitation efficiency, it is essential to know the field configuration of the wanted mode of propagation. These fields must be a solution to Maxwell's equations and must satisfy boundary conditions at the guiding interface. At the exciting end these fields must, at least approximately, match the fields impressed by the launching device. For many periodic structures, it is relatively straightforward to formulate the total field

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[1] F. D. Borgnis and C. H. Papas, "Electromagnetic waveguides," in "Encyclopedia of Physics," vol. 16, Springer-Verlag, Berlin, Germany,; 1958.

[2] L. Brillouin, "Wave guides for slow waves," *J. Appl. Phys.*, vol. 19, pp. 1023–1041; 1948.

[3] F. J. Zucker, "The guiding and radiation of surface waves," *Proc. Symp. on Modern Advances in Microwaves Techniques*, Polytechnic Institute of Brooklyn, N. Y.; 1954.

in the form of a sum of plane surface-wave modes, with the source field superimposed. For example, this procedure is followed in the case of the periodically iris-loaded waveguide[4] and open structures, such as corrugated plane and cylindrical surfaces.[5,6] It also applies to arrays of disks of various shapes.[7]

In obtaining the field solution for periodic structures, we use, with more or less justification, a theorem named after Floquet.[8] Following this theorem, we assume that on any periodic structure with, say, a periodicity b in the direction of the z coordinate, the field solution is itself periodic in z with the periodicity b of the structure. Taking a Fourier expansion, one obtains a field expression in the form of an infinite sum of "space harmonics." The propagation constant of the mth harmonic is given by

$$\beta_m = \beta_0 + \frac{2\pi m}{b}, \tag{1}$$

where m can be any positive or negative integer. We then have a sum of generally slow waves with decreasing phase velocity as the order m is increasing. On this sum of propagating modes we impose boundary conditions, namely, that the parallel electric field must vanish at a conducting surface, and that at the interface between the guiding structure and the rest of space the tangential components of the electromagnetic field are continuous. Since we have an infinite number of modes which are necessary to satisfy boundary conditions, we usually have an infinite set of equations to solve. One may, however, introduce suitable approximations and obtain results which are more or less meaningful.

A periodic array of conducting cylinders (Fig. 1) does not easily yield to this treatment. All one knows is that the tangential electric field vanishes on the surface of the elements making up the array, and also, if the cylinders are fairly thin, one may assume that all the currents flow in the direction of the cylinder axes. We do not know the current distribution on each of the elements of the array. We may, of course, use King's theory for an array of coupled antennas[9] once we specify driving conditions, but this procedure becomes hopelessly laborious for an array consisting of more than a very small number of elements. The case of a small number of elements (*i.e.*, short line) is not of great interest when one examines the characteristics of an array used as a transmission line, not as a terminated radiator. On the other hand, this type of periodic structure has already been used very extensively in Yagi antennas, and in the absence of a rigorous electromagnetic field solution we may at least examine experimentally the use of such structures as transmission lines, that is, as a means of guiding and transmitting power from point A to point B. This is the main objective of this report.

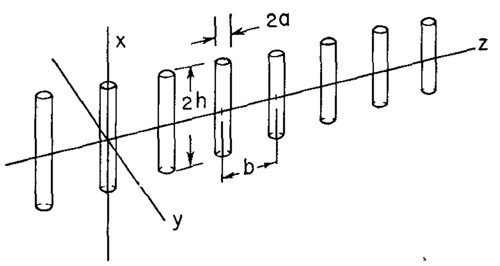

Fig. 1—Array of conducting cylinders.

II. Approximate Solutions for Phase Velocities

One may examine every possible wave mode on the structure by taking the analogy of a two-wire transmission line (Fig. 2). The lossless elements are represented by reactive impedances jX periodically shunted across the line. This analogy yields a propagation constant given by[10]

$$\cos \beta b = \cos kb + \frac{Z_0}{2X} \sin kb \tag{2}$$

where

$$k = \frac{2\pi}{\lambda_0} \qquad \beta = \frac{2\pi}{\lambda_g}.$$

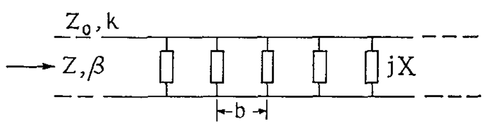

Fig. 2—Transmission line shunted by reactive elements jX.

This is a rather crude approach, but it yields some useful results within a limited range of parameters. In particular, it indicates a series of stop and pass bands as we change frequency. For small spacing of the elements when $b/\lambda_0 \ll 1$, (2) may be approximated by

$$\frac{k}{\beta} = 1 + \left(\frac{Z_0}{4\pi X}\right)\frac{\lambda}{b}. \tag{3}$$

This simple approximate dispersion relationship is quite consistent with experimental data.[11]

[4] E. L. Chu and W. W. Hansen, "The theory of disk-loaded waveguides," *J. Appl. Phys.*, vol. 18, pp. 996–1008; 1947.

[5] R. S. Elliot, "On the theory of corrugated plane surfaces," IRE Trans. on Antennas and Propagation, vol. AP-2, pp. 71–81; April, 1954.

[6] H. E. M. Barlow and A. E. Karbowiak, "An experimental investigation of the properties of corrugated cylindrical surface waveguides," *Proc. IEE*, vol. 101, pt. III, pp. 182–188; May, 1954.

[7] J. Shefer, "Plane Waves on a Periodic Structure of Circular Disks and their Application to Surface-Wave Antennas," Cruft Lab., Harvard University, Cambridge, Mass., Sci. Rept. No. 11; Ser. 2; December, 1961.

[8] L. Brillouin, "Wave Propagation in Periodic Structures," McGraw-Hill Book Co., Inc., New York, N. Y.; 1946.

[9] Ronold King, "Linear arrays: currents, impedances, and fields, I," IRE Trans. on Antennas and Propagation, vol. AP-7, pp. S440–S457; December, 1959.

[10] S. Silver, "Microwave Antenna Theory and Design," McGraw-Hill Book Co., Inc., New York, N. Y.; 1949.

[11] J. O. Spector, "An investigation of periodic structures for Yagi aerials," *Proc. IEE*, vol. 105, pt. B, pp. 38–44; January, 1958.

A more rigorous attempt to obtain a solution for phase velocities on a periodic cylinder structure is due to Dunbar, *et al.*,[12] who generalized some of Storer's[13] results for a single linear element. Referring to Fig. 3, $(E_x)_0$ is the x component of the electric field on an arbitrarily chosen reference element $n=0$ in an infinite array $n=\pm\infty$, due to the current elements $I_n(x')$. This tangential field is given by

$$(E_x)_0 = \frac{\eta}{4\pi j}\sum_n \int_{-h}^{+h} I_n(x')K_{n0}(x-x')dx' \cdots \quad (4)$$

where:

$$K_{n0}(x-x') = \left[1+\frac{\partial^2}{\partial x^2}\right]\frac{e^{-jkR_{n0}}}{R_{n0}},$$

$$R_{n0}^2 = (x-x')^2 + (a+nb)^2,$$

$$k = \frac{2\pi}{\lambda_0}; \quad \eta = 120\pi \text{ ohms}; \quad b \gg a.$$

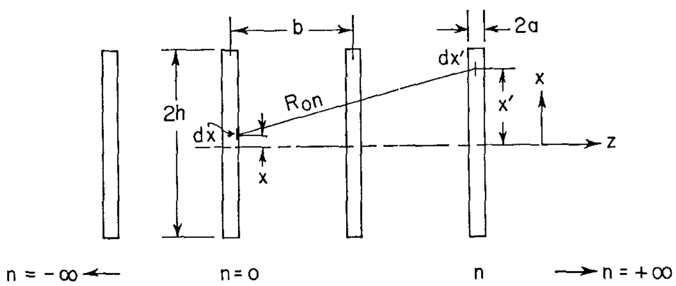

Fig. 3.

By defining a surface impedance

$$Z = -\frac{(E_x)_0}{I_0(x)}, \quad (5)$$

and multiplying each side of (4) by $I_n(x)$ integrating over the cylinder, we obtain

$$Z\int_{-h}^{+h} I_0^2(x)dx = \frac{j\eta}{4\pi}\sum_n \int_{-h}^{+h}\int I_0(x)I_n(x')K_{n0}(x-x')dx\,dx' \cdots \quad (6)$$

We assume further that the current distribution is the same in all elements apart from a phase shift which is related to the propagation of a slow wave along the structure. If this wave propagates with constant

$$\beta = \frac{2\pi}{\lambda_g} = \frac{\omega}{V_{ph}}$$

then

$$I_n(x) = I_0(x)e^{-j\beta nb} \quad (7)$$

and substitution in (6) yields

$$Z = \frac{j\eta}{4\pi}\frac{\sum_n e^{-j\beta nb}\int_{-h}^{+h}\int I_0(x)I_0(x')K_{n0}(x-x')dx\,dx'}{\int_{-h}^{+h} I_0^2(x)dx}. \quad (8)$$

This expression is stationary with respect to arbitrary variations in the current distribution $I_0(x)$. A trial function of the form

$$I_0(x) = \sin k(h-|x|) + C[1-\cos k(h-|x|)] \cdots \quad (9)$$

is chosen and substituted into (8), and requiring that

$$\frac{dZ}{dC} = 0$$

yields an expression for Z as a function of β, k, a, b, and h. One can now solve for β by imposing the condition that Z vanish on the cylinder surface. Experimental verification over a limited range of parameters has also been given.

The assumption that all the elements in the array have the same current distribution is somewhat questionable. Indeed, King[9] has shown that for a finite array of coupled elements, the current distribution varies considerably from one element to another. If the array is infinite in length, it may be argued that for reasons of symmetry all the currents must be equal, since we cannot specify any preferred location on an infinite array. By the same token, however, it may be argued that on any infinite physical structure, which must have some losses, all the currents are equal to zero at an infinite distance from the sources.

III. Experimental Determination of Transmission Characteristics

A block diagram of the experimental setup is given in Fig. 4. The insertion loss of the periodic structure, including matching units, was determined by comparing outputs of matched detectors 1 and 3. Reflected power was measured by detector 2. For more accurate determination of insertion loss, comparison was made by means of the same crystal in detector 3 by removing the structure under test from the circuit, at the same time also observing any change in reflected power by means of detector 2.

The transmitted power as a function of frequency on a periodic structure without matching unit is shown in Fig. 5. At frequencies below cutoff we observe a series of peaks spaced at intervals of frequency which increase with frequency. This shows that we have a resonant dispersive line. It is, therefore, necessary to introduce

[12] A. S. Dunbar, W. S. Lucke, E. M. T. Jones, and D. K. Reynolds, "Ridge and Corrugated Antenna Studies," Stanford Res. Inst., Stanford, Calif., Project No. 199, Final Rept; January, 1961.

[13] J. Storer, "Variational Solution to the Problem of the Symmetrical Cylindrical Antenna," Cruft Lab., Harvard University, Cambridge, Mass., Tech. Rept. No. 101; February, 1950.

matching networks at the transition between waveguide and periodic structure. A tapered periodic structure was found to be the most effective means for matching. In Fig. 6 we have the transmission characteristic for the same structure with tapered sections interposed. The power transmitted along the line is relatively constant over a band of 1200 Mc. In Fig. 7 the received power in detector 3 is compared with a sample of incident power as measured at detector 1 and reflected power at detector 2. The average insertion loss of this particular structure amounted to 2.5 db. For a number of other combinations of element length and spacing, we obtain values of insertion loss ranging from 2 to 4 db. Results for a 10.8-cm long line are summarized in Fig. 8(a)–(e).

The next step is to determine whether the transmission losses are caused by radiation and reflection from the terminations at both ends, or whether there exists any appreciable radiation loss from points along the periodic structure. In order to determine how transmission loss depends on the length of the line, we compare transmission loss of three structures of varying length (Fig. 9). The number of cylinder elements is 21, 51, 101, with a constant spacing and cylinder length. We note that there is almost no change in the amplitude of the received wave at detector 3, indicating that losses from points along the structure are by an order of magnitude smaller than losses that occur at the terminals. This result is consistent with other data from experimental work on Yagi antennas.[11]

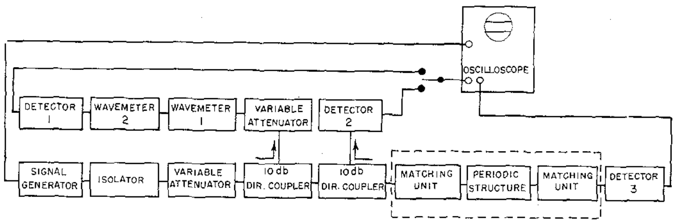

Fig. 4—Block diagram of experimental setup.

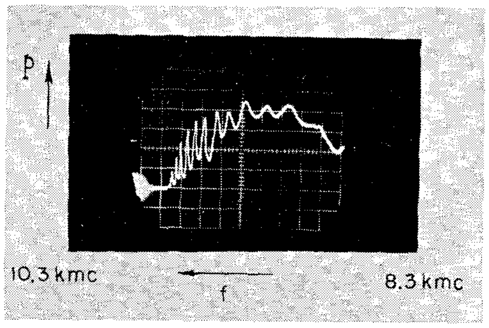

Fig. 5—Transmission characteristic of linear array. No matching network. $2h=12$ mm, $b=7.6$ mm, $N=15$.

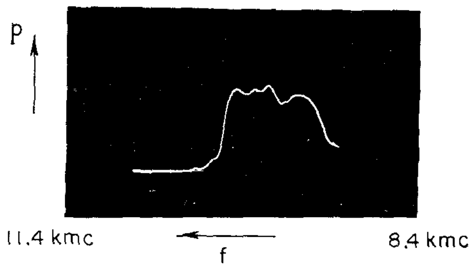

Fig. 6—Transmission characteristic of linear array with matching units at launching and receiving ends. $2h=12$ mm, $b=7.6$ mm, $N=15$.

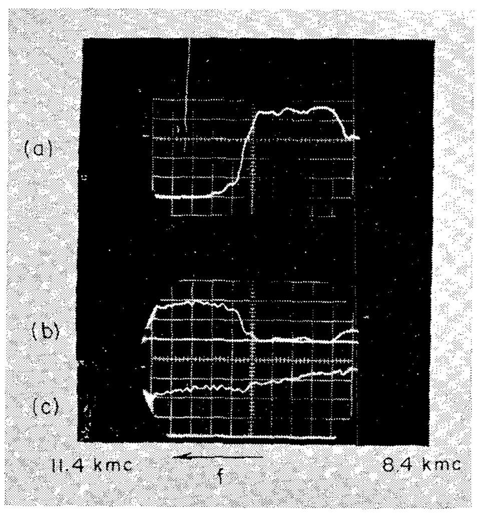

Fig. 7—Transmission characteristic of linear array. (a) Transmitted power. (b) Reflected power. (c) Sample of forward power in exciting waveguide. $2h=12$ mm, $b=2.54$, $N=30$.

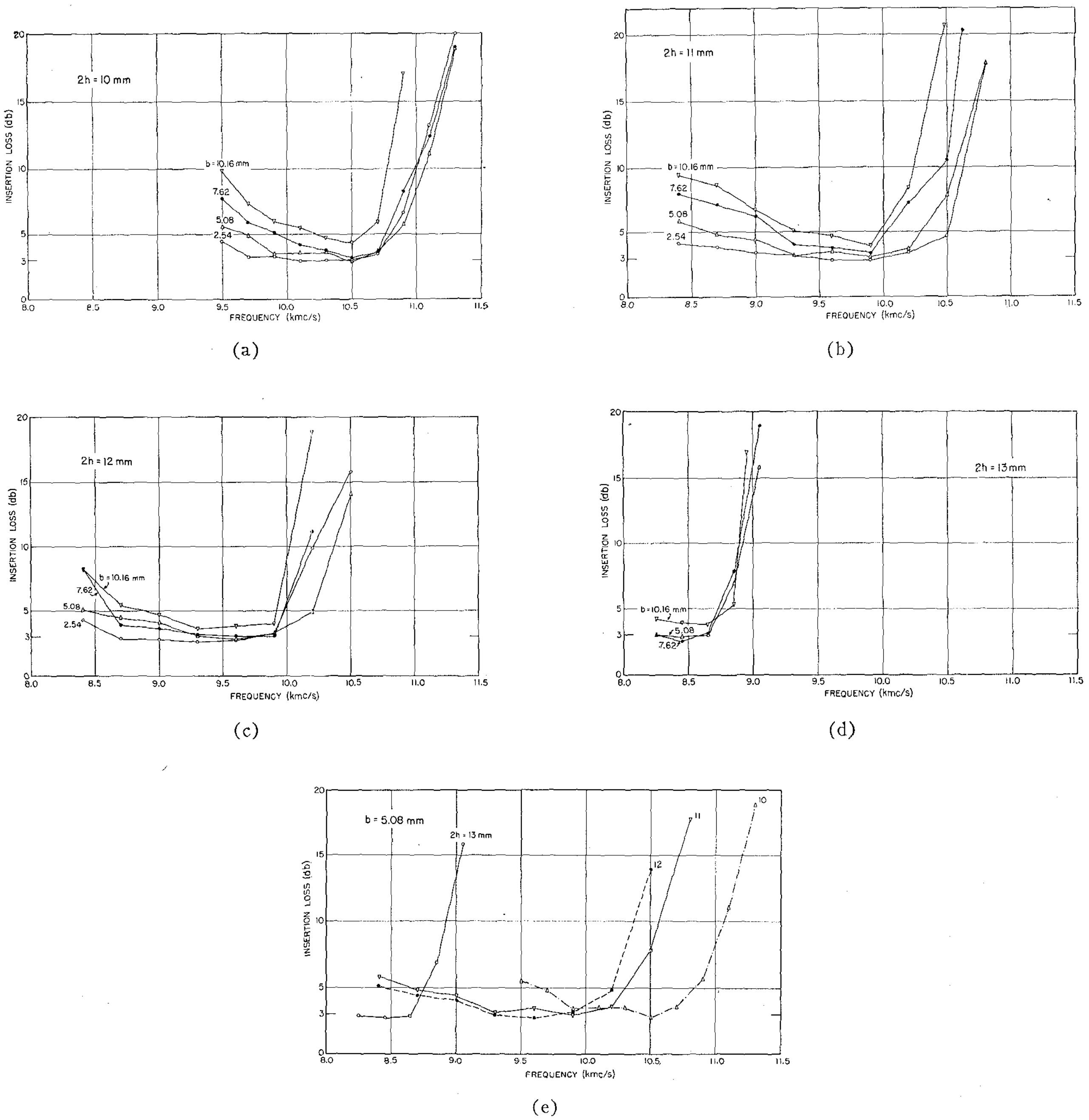

Fig. 8—Insertion loss of periodic structures, $L=10.8$ cm: (a) $2h=10$ mm, (b) $2h=11$ mm, (c) $2h=12$ mm, (d) $2h=13$ mm, (e) $b=5.08$ cm.

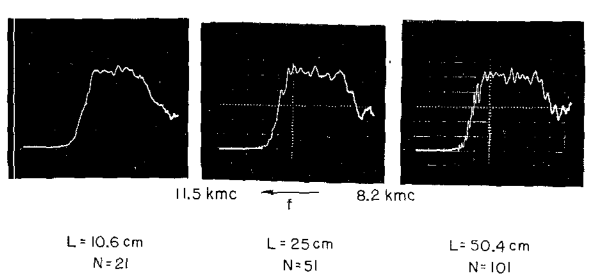

Fig. 9—Transmission on linear array vs length of array. $2h=11$ mm, $b=5.08$ mm.

IV. Nonuniform Lines

Bends and twists in the line were the two nonuniformities which were investigated. The bend in the periodic structure was made in a plane perpendicular to the plane containing the cylinder axes, so that the elements stayed parallel to one another. The line was made to go through a total angle of 180°, with a radius of curvature ranging between 1.2λ and 5λ. The effect of such a bend on transmission characteristics is shown in Fig. 10. We note that in going through a half-circle we lose some bandwidth. The band of uniform transmission is now 400-Mc wide, compared with 1200 Mc for a straight-line array. Still, it is noticeable that over a somewhat limited frequency band we may reverse the direction of the transmission line without losing too much power through radiation. This is in marked contrast to what we find in most other open waveguide structures. The cylinders stay parallel to one another in going through the bend, so that adjacent elements, which interact most strongly, are not much affected by a bend in the line.

The change in transmission characteristics where the plane of polarization is going through a 90° twist is shown in Fig. 11. We notice some deterioration at the edges of the pass band with almost no loss at mid band. This loss of bandwidth is more severe when the spacing between elements is increased from 2.5 mm to double that value, as shown in Fig. 12.

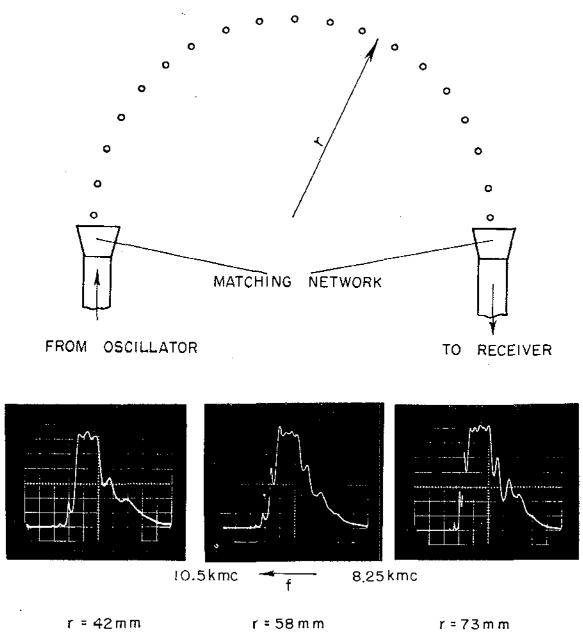

Fig. 10—Transmission on circular array vs radius of curvature. Total angle 180°. $2h=12$ mm. $b=5.08$ mm.

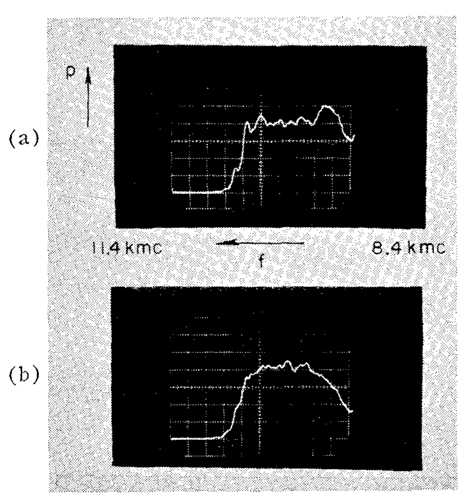

Fig. 11—Transmission on twisted array compared with array of parallel cylinders. Twist angle 90°. (a) Uniform line. (b) Twisted line. $2h=11$ mm, $b=2.54$ mm, $L=10.6$ cm.

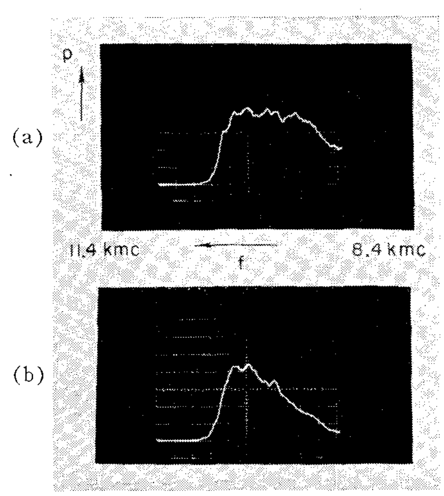

Fig. 12—Transmission on twisted array compared with array of parallel cylinders. Twist angle 90°. (a) Uniform line. (b) Twisted line. $2h=11$ mm, $b=5.08$mm, $L=10.6$ cm.

V. Phase Velocity

In order to measure phase velocity, the periodic structure is terminated by short circuits at both ends. By virtue of the periodicity of ω as a function of β, it can easily be shown that the number of resonances (*i.e.*, when the reasonator is an integral number of guided half-wavelengths long) in any one pass band is finite, and is equal to the number of periodic sections in the resonant transmission line.[14] The transmission characteristics for a periodic structure terminated by full-wave cylinders are shown in Fig. 13. Every peak of transmission can thus be associated with a known number of half-wavelengths in the resonator standing wave pattern, and, hence, the propagation constant is determined. A set of typical results is shown in Fig. 14. The accuracy of this method depends on the quality of the terminating elements and the accuracy in measuring the effective resonator length. Accuracies of 10^{-3} are easily obtained.

VI. Conclusions

It has been shown that an open structure of an array of parallel conducting cylinders can be used as a transmission line. Losses are effectively independent of the length of the line, and we may bend or twist the line without radiation loss over a wide band of frequencies. This type of transmission line may have certain advantages over hollow metal pipes under circumstances where a saving in weight is of importance, since the supporting cylinders can be made thin enough to have very small weight and may be embedded in a lightweight medium, such as styrofoam. There may also be an advantage in not requiring physical contact of conducting metal along the line. The line may be broken up into several building blocks, and coupling between sections may be controlled by changing the proximity of the blocks to one another. This may provide a convenient way for experimentally optimizing certain microwave systems.

[14] J. C. Slater, "The Design of Linear Accelerators," Res. Lab. for Electronics, Mass. Inst. Tech., Cambridge, Mass. Tech. Rept. No. 47; September, 1947.

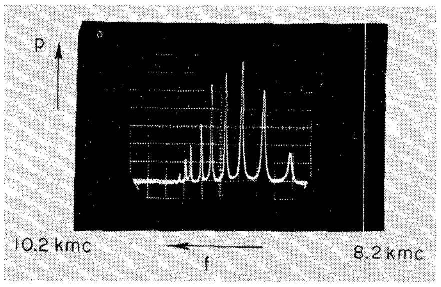

Fig. 13—Transmission on resonant line. $2h = 13$ mm, $b = 5.08$ mm, $L = 10.6$ cm.

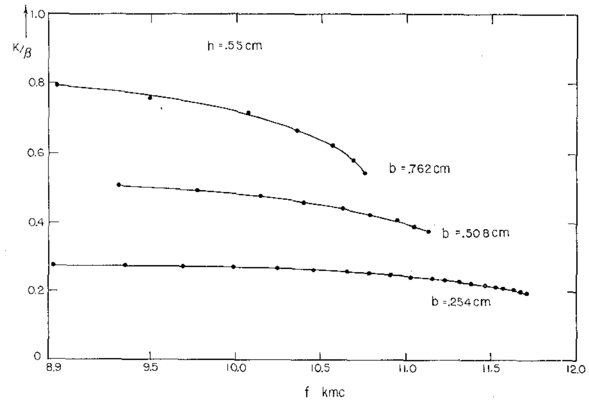

Fig. 14—Phase velocity on uniform array. $2h = 11$ mm.

Acknowledgment

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